Generation of photonic graph states - part II

Yuan Zhan
Department of Physics, University of Colorado Boulder
yuan.zhan@colorado.edu

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1 Introducing feedback to the generation of graph states

From our discussion in part I, we know that we are able to generate many important graph states, like Greenberger-Horne-Zeilinger (GHZ) states, cluster states, repeater graph states (RGSs) and tree states with the help of an ancilla, in addition to a quantum emitter [1]. Experimentally, we usually choose quantum dots or color centers as our emitter and ancilla. However, due to the inhomogeneity in energy-level structure between different quantum dots or color centers, it will make things much easier if we could use only one single quantum emitter to generate the graph states we want. This is possible if we introduce time-delayed feedback into our procedure [2] [3].

1.1 Generation of 2D cluster states via time-delayed feedback

H. Pichler et.al. proposed a method to generate 2D cluster states with a square-lattice structure via time-delayed feedback for universal quantum computation [2]. Actually, more generally, we can use this feedback technique to generate other graph states like RGSs and tree states, which we will discuss in the next two sections, and even arbitrary graph states according to [3] by carefully designing the scattering S-matrix. Let's review the proposal in [2] firstly, and see what ideas we can borrow for our purpose.

The idea is that via time-delayed feedback, we can make photons re-scatter with the emitter (the first-time scattering happens at the time when the emitter generates this photon). We have known that we can easily generate 1D cluster states and GHZ states using only one quantum emitter [4], and feedback can realize the entanglement in the other dimension, so that generate 2D cluster states. Physically, we can realize this feedback procedure using an atom-like quantum emitter coupled to a 1D waveguide with a distant mirror, as shown in Figure 1(a). The 1D cluster states are characterized by so-called matrix product states (MPSs), and the 2D states realized via feedback are characterized by projected entangled pair states (PEPSs) [2].

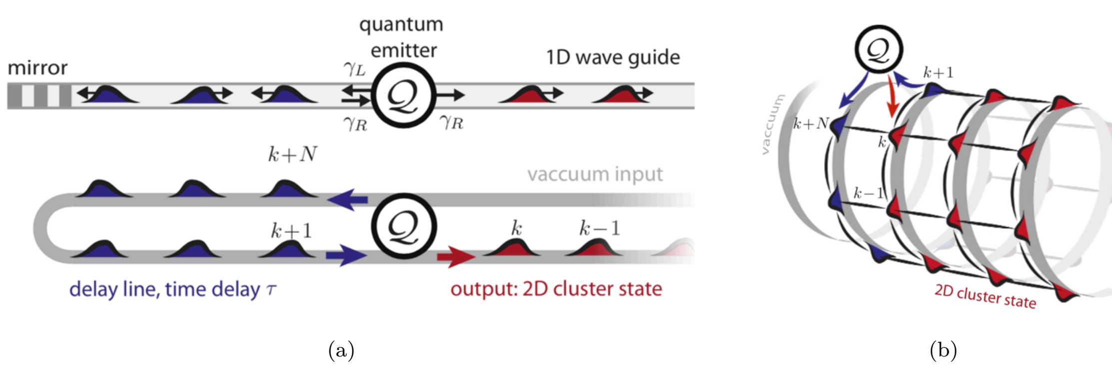


Figure 1: (a) A quantum emitter coupled to a 1D waveguide system with a distant mirror for generation of 2D cluster states via time-delayed feedback. (b) The resulting entanglement structure.

1.1.1 Physical system

The required physical system is described as follows [2]. The emitter $\mathcal{Q}$ used has a energy-level structure as shown in Figure 2, who has two metastable states $|g_1\rangle$ and $|g_2\rangle$ which can be coherently manipulated by a field $\Omega_1(t)$. The state $|g_2\rangle$ is coupled to an excited state $|e_L\rangle$ via a field with Rabi frequency $\Omega_2(t)$, and after excitation, the atom can decay back to $|g_2\rangle$ and emit a photon into the waveguide. We assume the emitted photons are unidirectional, and are directed towards the mirror (to the left in Figure 1(a)). There is another excited state, $|e_R\rangle$, which is degenerate with $|e_L\rangle$ and coupled to the reflected photon. We denote the corresponding decay rates by γ_L and γ_R.

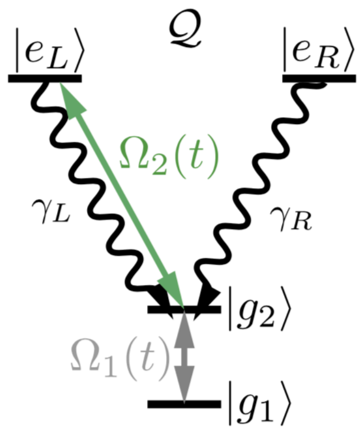


Figure 2: Energy-level structure of the emitter.

In the language of quantum information, we denote $|g_1\rangle \equiv |0\rangle_{\mathcal{Q}}$ and $|g_2\rangle \equiv |1\rangle_{\mathcal{Q}}$. For photons, we encode the absence of a photon in the k-th pulse as $|0\rangle_k$, and presence as $|1\rangle_k$. This encoding of photons is worth our attention, since usually we use polarizations to encode photonic qubits, which enables direct single-qubit or controlled-gate operations on photons. Using the absence and presence of a photon to encode it may limit our operation choices. This problem can be solved by using a modified cavity QED system, which will be discussed later.

1.1.2 Protocol

The detailed description of the protocol is as follows [2].

Firstly, we need to generate a 1D cluster state of left-propagating photons. We initially prepare the atom (emitter) in the state $|0\rangle_{\mathcal{Q}}$, i.e. $|g_1\rangle$. Then, we apply a $\pi/2$ pulse of $\Omega_1(t)$ on the atom, followed by a π pulse of $\Omega_2(t)$ to excite the transition $|g_2\rangle \rightarrow |e_L\rangle$. The subsequent decay results in an emitted photon, which is entangled with the atom. Repeating this process ($\Omega_1(\pi/2) \rightarrow \Omega_2(\pi) \rightarrow$decay), we can have a 1D cluster state, similar to the proposal of [4], as described in 2.2.2 of part I [1]. A mathematical description is as follows.

$$\begin{aligned} &|0\rangle_{\mathcal{Q}} \xrightarrow{\Omega_1(\frac{\pi}{2})} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{\mathcal{Q}} \xrightarrow{\Omega_2(\pi)} \frac{1}{\sqrt{2}}(|0\rangle + |e_L\rangle)_{\mathcal{Q}} \xrightarrow{\text{decay}} \frac{1}{\sqrt{2}}(|0\rangle_{\mathcal{Q}}|0\rangle_1 + |1\rangle_{\mathcal{Q}}|1\rangle_1) \\ &\xrightarrow{\Omega_1(\frac{\pi}{2})} \frac{1}{\sqrt{2}}(|+\rangle_{\mathcal{Q}}|0\rangle_1 + |-\rangle_{\mathcal{Q}}|1\rangle_1) = CZ_{\mathcal{Q},1}|+\rangle_{\mathcal{Q}}|+\rangle_1 \\ &\xrightarrow{\cdots\cdots} \text{1D cluster state.} \end{aligned} \tag{1}$$

Then, we need to consider feedback, i.e. the re-scattering of right-propagating photons with the emitter. When a right-propagating photon arrives at the emitter again, if the emitter is in state $|g_2\rangle$, then the right-propagating photon can be resonantly coupled to the $|g_2\rangle \rightarrow |e_R\rangle$ transition, so that pick up a phase shift of π without reflection [5]. If the emitter is in state $|g_1\rangle$, or the photon is in state $|0\rangle_k$ (no photon actually), there is no interaction. So the interaction between the emitter and the right-propagating photon is actually a controlled-phase gate, where the emitter plays the role of control bit,

$$CZ_{\mathcal{Q},k} = |0\rangle_{\mathcal{Q}}\langle 0| \otimes \mathbb{1}_k + |1\rangle_{\mathcal{Q}}\langle 1| \otimes \sigma_k^z, \tag{2}$$

where σ_k^z represents the single-qubit Z gate on k-th photon (pulse). So this $CZ_{\mathcal{Q},k}$ gate entangles the emitter $\mathcal{Q}$ and k-th photon.

After the re-scattering happens, the emitter will subsequently generate another photon, and this photon will inherit this entanglement, so that result in a 2D entanglement structure, as shown in Figure 1(b).

For the last step, we need to carefully design the sequence. If we define the time interval between two adjacent pulses as T, the time delay of the feedback line with a time delay of τ, and N pulses for each round (number of qubits for each row in the resulting square lattice), then we can design the feedback so that $\tau = (N - \frac{3}{4})T$ for example (they choose $\tau = (N - \frac{1}{2})T$ in [2]). For each step, the emitter generates the $(k + N - 1)$-th pulse (with or without a photon), then interacts with the k-th reflected pulse after $T/4$, and generates the $(k + N)$-th pulse after another $3T/4$, as shown in Figure 1(a).

From our former discussion, we know that in the generation of 1D cluster state string, we do: $[\Omega_1(\pi/2) \rightarrow \Omega_2(\pi) \rightarrow$decay], for each step. Now after introducing feedback and the entanglement in another dimension, the sequence becomes: $[\Omega_1(\pi/2) \rightarrow \mathcal{Q}$ interacts with the reflected pulse $\rightarrow \Omega_2(\pi) \rightarrow$decay]. The $\Omega_1(\pi/2)$ pulse is equivalent to a Hadamard gate applied on the emitter, $H_{\mathcal{Q}}$, the interaction between $\mathcal{Q}$ and reflected pulse is equivalent to a controlled-phase gate, $CZ_{\mathcal{Q},k}$, and the $\Omega_2(\pi)$ pulse is equivalent to a controlled-X gate ($CNOT$ gate), $CX_{\mathcal{Q},k+N}$. The last $CX_{\mathcal{Q},k+N}$ gate can be easily verified, since from our definition, before $\mathcal{Q}$ generates the $(k+N)$-th pulse, the $(k+N)$-th pulse is in $|0\rangle_{k+N}$ state (because there is no photon in $(k+N)$-th pulse), and if $\mathcal{Q}$ is in $|g_1\rangle = |0\rangle_{\mathcal{Q}}$ state, it cannot generate a photon, leaving the $(k+N)$-th pulse in $|0\rangle_{k+N}$ state, while if $\mathcal{Q}$ is in $|g_2\rangle = |1\rangle_{\mathcal{Q}}$ state, it will change the $(k+N)$-th pulse to $|1\rangle_{k+N}$ state, which is exactly the function of a $CNOT$ gate with $\mathcal{Q}$ being the control bit. So formally, in order to generate a $M \times N$ square lattice, the initial state of the emitter $\mathcal{Q}$ and all the photons is $|0\rangle_{\mathcal{Q}} \otimes_{M\times N} |0\rangle_{\text{photon}}$, i.e. the product state of $\mathcal{Q}$ and all $(M \times N)$ photons, then the resulting 2D cluster state can be expressed mathematically as below,

$$|\psi_{\text{2D cluster}}\rangle = \underbrace{M_{\mathcal{Q}}}_{\text{disentangle } \mathcal{Q}} H_{\mathcal{Q}} \underbrace{\left(\sum_{k=1}^{(M-1)\times N} CX_{\mathcal{Q},k+N} CZ_{\mathcal{Q},k} H_{\mathcal{Q}}\right)}_{\text{following rows including re-scattering}} \underbrace{\left(\sum_{k=1}^{N} CX_{\mathcal{Q},k} H_{\mathcal{Q}}\right)}_{\text{first row}} \underbrace{\left(|0\rangle_{\mathcal{Q}} \otimes_{M\times N} |0\rangle_{\text{photon}}\right)}_{\text{initial state}}. \tag{3}$$

There is one point worth noticing that this 2D cluster state has a shifted periodic boundary condition, which means the right end of the upper row is entangled with the left end of the lower row, as shown in Figure 3.

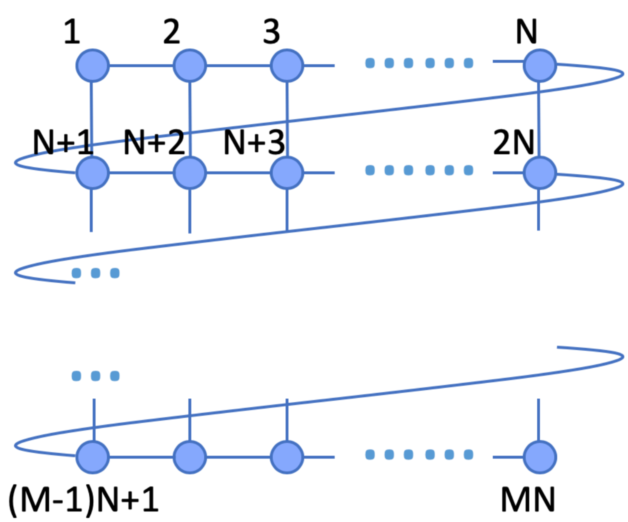


Figure 3: 2D cluster state with a shifted periodic boundary condition.

To better understand and verify this process, we can consider a simple 2×2 example (so $M = N = 2$).

The whole process can be mathematically described as below.

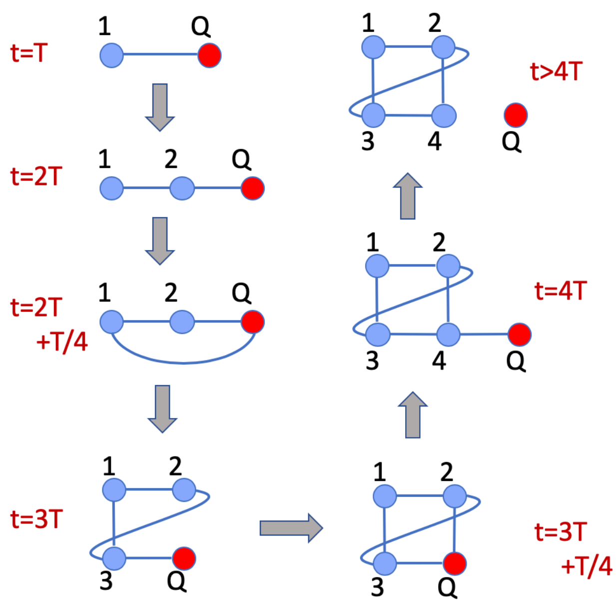
$$
\begin{aligned}
&|0\rangle_{\mathcal{Q}} \xrightarrow[\Omega_1(\frac{\pi}{2}),H_{\mathcal{Q}}]{t=0} \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{\mathcal{Q}} \xrightarrow[\Omega_2(\pi),CX_{\mathcal{Q},1}]{t=\frac{T}{2}} \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{\mathcal{Q}1}\\
&\xrightarrow[\Omega_1(\frac{\pi}{2}),H_{\mathcal{Q}}]{t=T} \frac{1}{2}(|00\rangle+|10\rangle+|01\rangle-|11\rangle)_{\mathcal{Q}1} \xrightarrow[\Omega_2(\pi),CX_{\mathcal{Q},2}]{t=T+\frac{T}{2}} \frac{1}{2}(|000\rangle+|110\rangle+|001\rangle-|111\rangle)_{\mathcal{Q}21}\\
&\xrightarrow[\Omega_1(\frac{\pi}{2}),H_{\mathcal{Q}}]{t=2T} \frac{1}{2\sqrt{2}}(|000\rangle+|100\rangle+|010\rangle-|110\rangle+|001\rangle+|101\rangle-|011\rangle+|111\rangle)_{\mathcal{Q}21}\\
&\xrightarrow[\text{re-scatter }\#1,CZ_{\mathcal{Q},1}]{t=2T+\frac{T}{4}} \frac{1}{2\sqrt{2}}(|000\rangle+|100\rangle+|010\rangle-|110\rangle+|001\rangle\underset{\triangle}{-}|101\rangle-|011\rangle\underset{\triangle}{-}|111\rangle)_{\mathcal{Q}21}\\
&\xrightarrow[\Omega_2(\pi),CX_{\mathcal{Q},3}]{t=2T+\frac{T}{2}} \frac{1}{2\sqrt{2}}(|0000\rangle+|1100\rangle+|0010\rangle-|1110\rangle+|0001\rangle-|1101\rangle-|0011\rangle-|1111\rangle)_{\mathcal{Q}321}\\
&\xrightarrow[\Omega_1(\frac{\pi}{2}),H_{\mathcal{Q}}]{t=3T} \frac{1}{4}(|0000\rangle+|1000\rangle+|0100\rangle-|1100\rangle+|0010\rangle+|1010\rangle-|0110\rangle+|1110\rangle\\
&\qquad+|0001\rangle+|1001\rangle-|0101\rangle+|1101\rangle-|0011\rangle-|1011\rangle-|0111\rangle+|1111\rangle)_{\mathcal{Q}321}\\
&\xrightarrow[\text{re-scatter }\#2,CZ_{\mathcal{Q},2}]{t=3T+\frac{T}{4}} \frac{1}{4}(|0000\rangle+|1000\rangle+|0100\rangle-|1100\rangle+|0010\rangle\underset{\triangle}{-}|1010\rangle-|0110\rangle\underset{\triangle}{-}|1110\rangle\\
&\qquad+|0001\rangle+|1001\rangle-|0101\rangle+|1101\rangle-|0011\rangle\underset{\triangle}{+}|1011\rangle-|0111\rangle\underset{\triangle}{-}|1111\rangle)_{\mathcal{Q}321}\\
&\xrightarrow[\Omega_2(\pi),CX_{\mathcal{Q},4}]{t=3T+\frac{T}{2}} \frac{1}{4}(|00000\rangle+|11000\rangle+|00100\rangle-|11100\rangle+|00010\rangle-|11010\rangle-|00110\rangle-|11110\rangle\\
&\qquad+|00001\rangle+|11001\rangle-|00101\rangle+|11101\rangle-|00011\rangle+|11011\rangle-|00111\rangle-|11111\rangle)_{\mathcal{Q}4321}\\
&\xrightarrow[\Omega_1(\frac{\pi}{2}),H_{\mathcal{Q}}]{t=4T} \frac{1}{4\sqrt{2}}(|00000\rangle+|10000\rangle+|01000\rangle-|11000\rangle+|00100\rangle+|10100\rangle-|01100\rangle+|11100\rangle\\
&\qquad+|00010\rangle+|10010\rangle-|01010\rangle+|11010\rangle-|00110\rangle-|10110\rangle-|01110\rangle+|11110\rangle\\
&\qquad+|00001\rangle+|10001\rangle+|01001\rangle-|11001\rangle-|00101\rangle-|10101\rangle+|01101\rangle-|11101\rangle\\
&\qquad-|00011\rangle-|10011\rangle+|01011\rangle-|11011\rangle-|00111\rangle-|10111\rangle-|01111\rangle+|11111\rangle)_{\mathcal{Q}4321}\\
&\xrightarrow[M_Q]{t>4T} \frac{1}{4}(|0000\rangle+|1000\rangle+|0100\rangle-|1100\rangle+|0010\rangle-|1010\rangle-|0110\rangle-|1110\rangle\\
&\qquad+|0001\rangle+|1001\rangle-|0101\rangle+|1101\rangle-|0011\rangle+|1011\rangle-|0111\rangle-|1111\rangle)_{4321}.
\end{aligned}
\tag{4}
$$

On the other hand, for a 2×2 2D cluster state with the form of Figure 3, we have

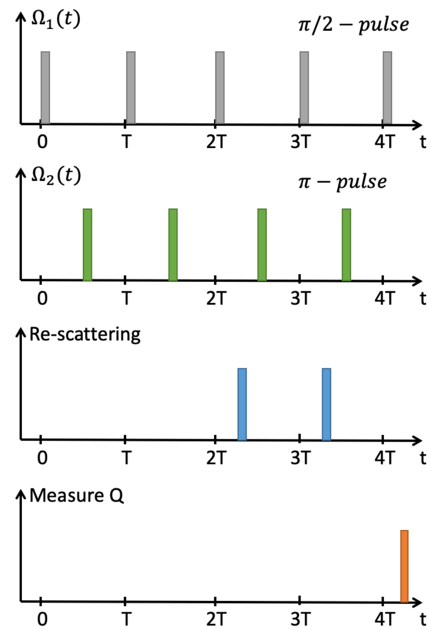
$$
\begin{aligned}
|\psi_{2\times 2\text{ 2D cluster}}\rangle &= CZ_{2,4}CZ_{3,4}CZ_{1,3}CZ_{2,3}CZ_{1,2}\,|++++\rangle_{4321}\\
&= \frac{1}{\sqrt{2}}CZ_{2,4}CZ_{3,4}CZ_{1,3}CZ_{2,3}\,|++\rangle_{43}\,(|0+\rangle+|1-\rangle)_{21}\\
&= \frac{1}{2\sqrt{2}}CZ_{2,4}CZ_{3,4}CZ_{1,3}CZ_{2,3}\,|+\rangle_4\,(|000\rangle+|001\rangle+|010\rangle-|011\rangle+|100\rangle+|101\rangle+|110\rangle-|111\rangle)_{321}\\
&= \frac{1}{2\sqrt{2}}CZ_{2,4}CZ_{3,4}CZ_{1,3}\,|+\rangle_4\,(|000\rangle+|001\rangle+|010\rangle-|011\rangle+|100\rangle+|101\rangle-|110\rangle+|111\rangle)_{321}\\
&= \frac{1}{2\sqrt{2}}CZ_{2,4}CZ_{3,4}\,|+\rangle_4\,(|000\rangle+|001\rangle+|010\rangle-|011\rangle+|100\rangle-|101\rangle-|110\rangle-|111\rangle)_{321}\\
&= \frac{1}{4}CZ_{2,4}(|0000\rangle+|0001\rangle+|0010\rangle-|0011\rangle+|0100\rangle-|0101\rangle-|0110\rangle-|0111\rangle\\
&\quad+|1000\rangle+|1001\rangle+|1010\rangle-|1011\rangle-|1100\rangle+|1101\rangle+|1110\rangle+|1111\rangle)_{4321}\\
&= \frac{1}{4}(|0000\rangle+|0001\rangle+|0010\rangle-|0011\rangle+|0100\rangle-|0101\rangle-|0110\rangle-|0111\rangle\\
&\quad+|1000\rangle+|1001\rangle-|1010\rangle+|1011\rangle-|1100\rangle+|1101\rangle-|1110\rangle-|1111\rangle)_{4321}.
\end{aligned}
\tag{5}
$$

We can see that the results in Eqn. 4 and Eqn. 5 are identical.

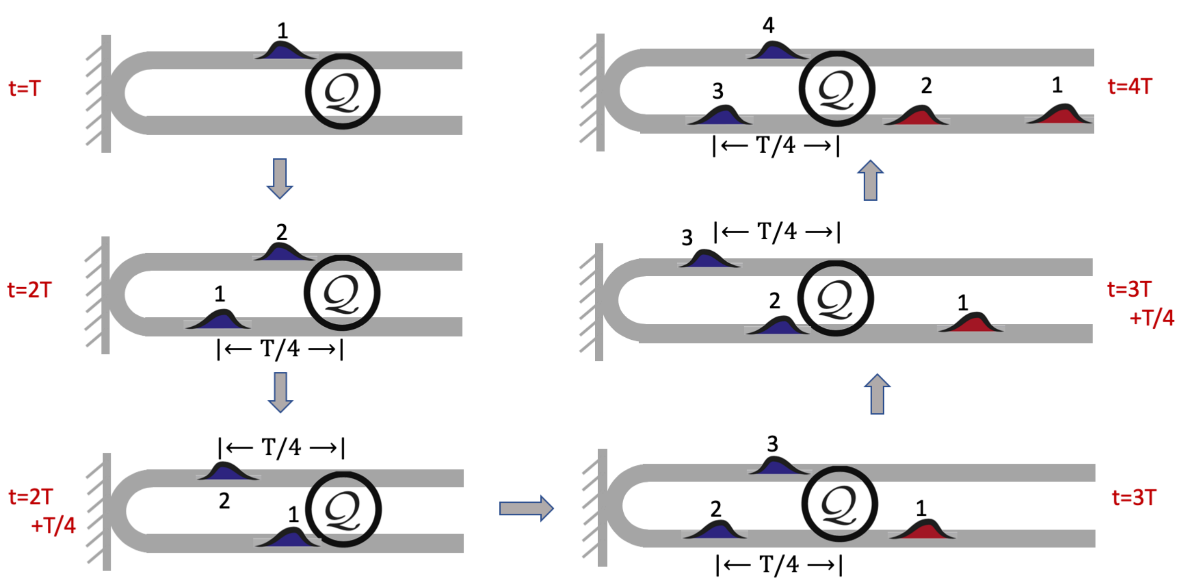
Schematically, this example can be expressed as below.



(a)



(b)



(c)

Figure 4: (a) Schematic expression of 2×2 2D cluster state generation. (b) Operation sequence. (c) The waveguide system with feedback.

1.2 An alternative physical system

We notice that using this physical system, it is not convenient to directly apply operations on photons, since $|0\rangle_{\text{photon}}$ means no photon. An alternative energy-level structure of the emitter is shown in Figure 5, where we could use polarizations to encode the photon qubits. In this case, $|g_1\rangle_{\mathcal{Q}}$ and $|g_2\rangle_{\mathcal{Q}}$ are coupled to $|e_L\rangle$ and $|e_R\rangle$ separately via $\Omega_2(t)$ pulse. The photon emitted by $|e_L\rangle \to |g_1\rangle$ transition is encoded as $|0\rangle_{\text{photon}}$, and by $|e_R\rangle \to |g_2\rangle$ transition as $|1\rangle_{\text{photon}}$.

We also require that when a reflected photon re-scatters with the emitter, $|0\rangle_k$ photon won't have a phase shift, while $|1\rangle_k$ photon will pick up a π phase shift, which realizes a controlled-phase gate.

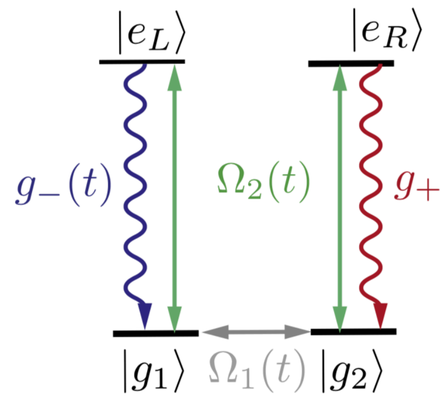


Figure 5: An alternative energy-level structure of the emitter.

1.3 Key techniques

As we have discussed so far, the key techniques in the feedback protocol are that:

- feedback realizes the second-time entanglement between the emitter and the re-scattering photon;
- the second-time entanglement between the emitter and the photon can be inherited by the photon generated subsequently;
- if we don't want the arriving reflected photon interact with the emitter for the second time, we can also make the emitter in state $|0\rangle_{\mathcal{Q}}$, by a single-qubit operation on the emitter.
- we need to carefully design the operation sequence, both on the emitter and the photons, and the length of time-delayed feedback.

2 Protocol for generating tree states using one quantum emitter

From what we have learned about the key feedback techniques in [2], and combining the ideas in part I [1], it is possible to generate tree states using only one quantum emitter.

As an example, we can see how to generate a tree state whose depth $d = 3$, and with $k = 4$ arms. The protocol is described as follows.

(a) Pump the emitter 4 times;

(b) Pump the emitter for the 5-th time;

(c) Apply Hadamard gates to the emitter $\mathcal{Q}$ and the 5-th photon, which swaps their positions;

(d) Measure $\mathcal{Q}$ to detach the emitter;

(e) Repeat (b)-(e) for four times;

(f) Let photon 1-4 pass the emitter again without interacting with the emitter, while let photon 5 interact with the emitter again;

(g) Repeatedly, let photon 6-9, 11-14, 16-19 pass without interaction, and entangle the emitter with photon 10, 15, and 20;

(h) Pump the emitter to generate the 21-th photon;

(i) Apply Hadamard gates to $\mathcal{Q}$ and the 21-th photon, so that the 21-th photon inherits all the entanglements of $\mathcal{Q}$, and then measure $\mathcal{Q}$ to detach it from the tree;

(j) Repeat (a)-(k) for four times;

(k) The emitter $\mathcal{Q}$ interacts with the reflected photons 21, 42, 63, and 84;

(l) Finally, the emitter generates the 85-th photon, and this photon inherits the entanglement relations of $\mathcal{Q}$. Then $\mathcal{Q}$ is measured and detached from the tree, leaving a $d = 3$, $k = 4$ tree state.

Additionally, we need to carefully design the length of the time-delayed feedback. It should be long enough so that the 21-th photon arrives at the emitter again after all the other three child trees are generated. Furthermore, the length of time-delayed feedback will increase exponentially with the increase of the depth of the state.

Mathematically, we can describe the protocol as follows.

$$\begin{aligned}|\psi_{\text{tree}}\rangle =& M_{\mathcal{Q}} H_{\mathcal{Q}} P_{\mathcal{Q},85} \left(\prod_{m=1}^{4} CZ_{\mathcal{Q},21m} \right) \\ & \prod_{l=0}^{3} \left[M_{\mathcal{Q}} H_{\mathcal{Q}} P_{\mathcal{Q},21(l+1)} \left(\prod_{k=1}^{4} CZ_{\mathcal{Q},21l+5k} \right) \prod_{j=0}^{3} \left(M_{\mathcal{Q}} H_{\mathcal{Q}} P_{\mathcal{Q},21l+5j+5} \prod_{i=1}^{4} (H_{21l+5j+i} P_{\mathcal{Q},21l+5j+i}) \right) \right] |0\rangle \,.\end{aligned} \tag{6}$$

It is much more intuitional to use schematic expression, as shown in Figure 6, Figure 7, and Figure 8.

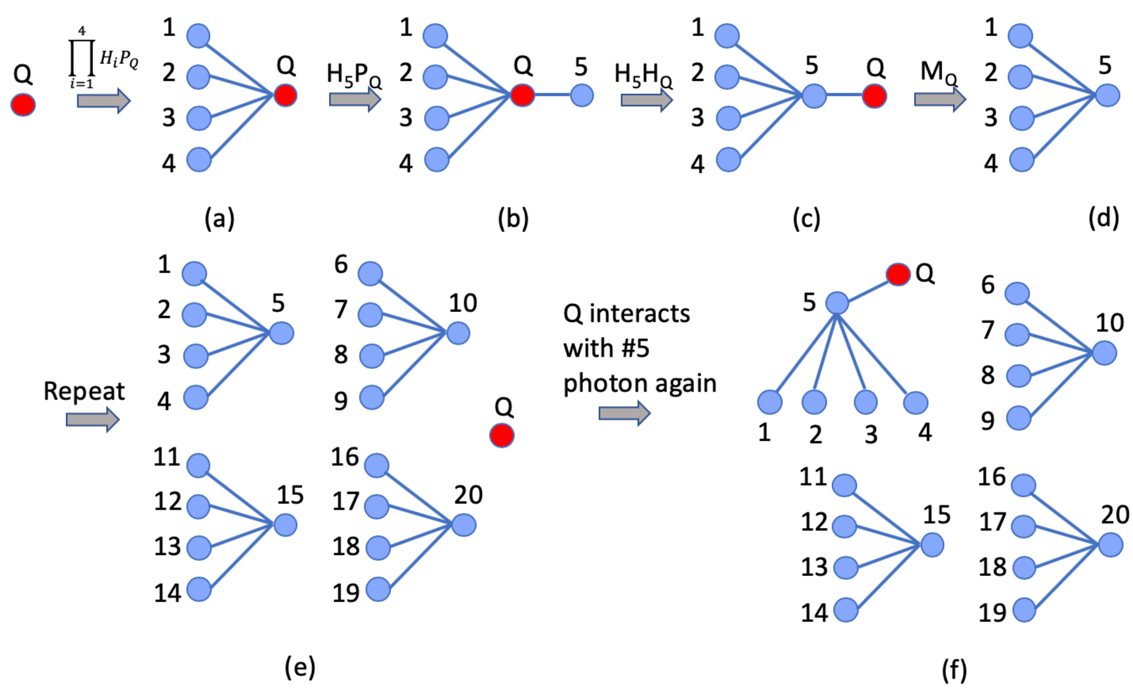


Figure 6: Schematic description of a $d = 3$, $k = 4$ tree state generate, (a)-(f).

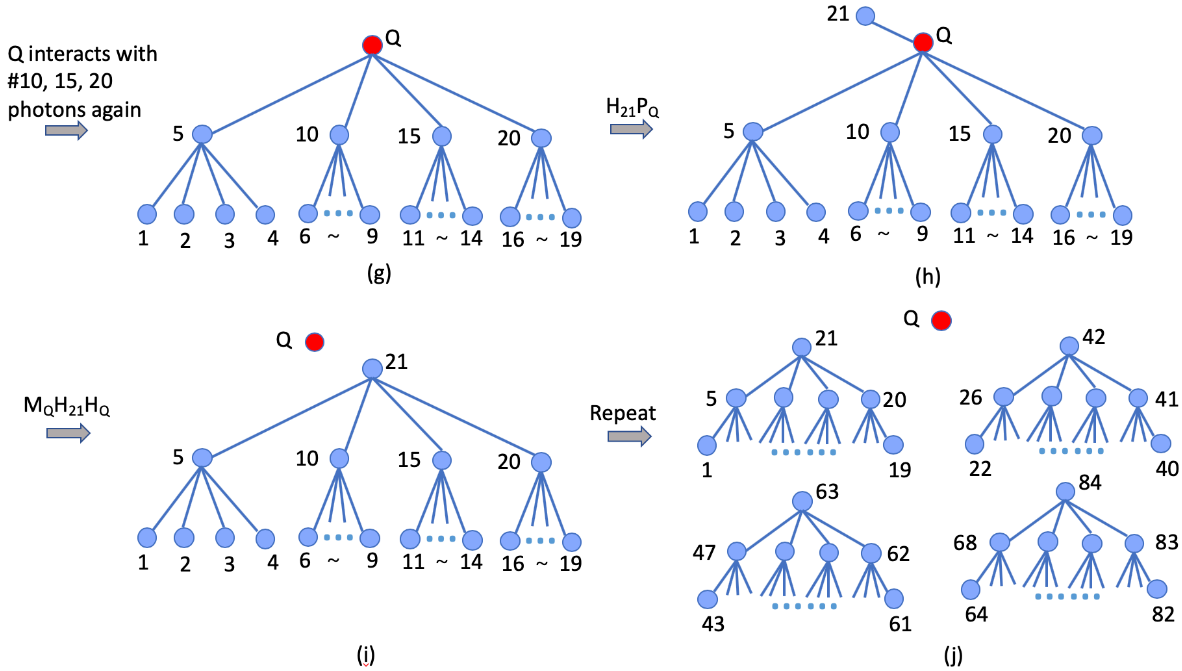


Figure 7: Schematic description of a $d = 3$, $k = 4$ tree state generate, (g)-(j).

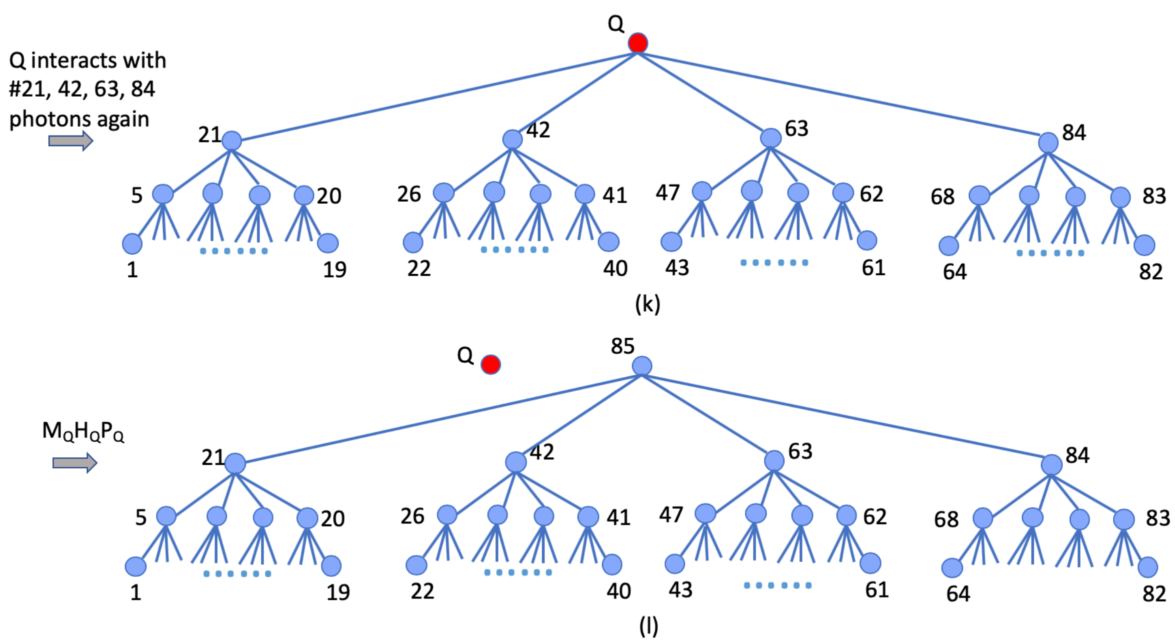


Figure 8: Schematic description of a $d = 3$, $k = 4$ tree state generate, (k)-(l).

3 Protocol for generating RGSs using one quantum emitter

As proposed in [6], photonic repeater states (RGSs) can be useful for all-photonic quantum repeater in quantum communication. The generation of RGSs using one quantum emitter with the help of an ancilla is proposed in [7]. However, if we introduce feedback, it is also possible to use only one quantum emitter to generate RGSs.

For example, the protocol for generation of a $N = 6$ RGS is described below.

(a) Pump the emitter 6 times;

(b) Do local complementation (LC) on the emitter $\mathcal{Q}$;

(c) Measure $\mathcal{Q}$ to detach it from the graph;

(d) The emitter $\mathcal{Q}$ interacts with the 1-st photon again, so that entangles with it;

(e) $\mathcal{Q}$ generated the 7-th photon;

(f) Apply Hadamard gates to $\mathcal{Q}$ and the 7-th photon, and then measure $\mathcal{Q}$ to detach it from the graph. The 7-th photon then becomes the arm for the 1-st photon;

(g) Repeat (d)-(f) to generate other 5 arms. The result is a $N = 6$ RGS.

Mathematically, we can describe this protocol as follows,

$$|\psi_{\mathrm{RGS}}\rangle = \prod_{m=1}^{6} \left(M_{\mathcal{Q}} H_{\mathcal{Q}} P_{\mathcal{Q},m+6} CZ_{\mathcal{Q},m}\right) M_{\mathcal{Q}} \left(e^{i(\pi/2)(Y+Z)/\sqrt{2}}\right)_{\mathcal{Q}} \prod_{n=1}^{6} \left[\left(e^{i(\pi/2)(X+Y)/\sqrt{2}}\right)_{n} H_n P_{\mathcal{Q},n}\right] |0\rangle \, . \quad (7)$$

Schematically, the protocol is shown in Figure 9.

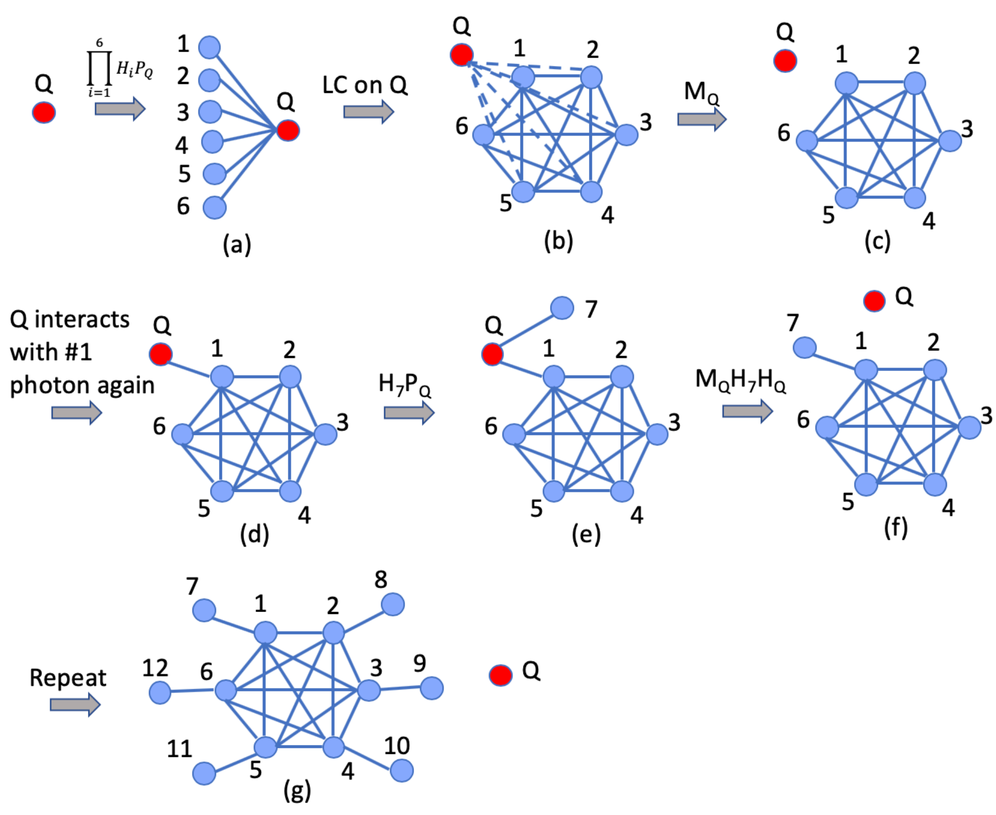


Figure 9: Schematic description of a $N = 6$ RGS.

4 Experimental requirements

To experimentally realize the protocols above, there are several requirements that need to be fulfilled.

- We need the emitter to have the desired energy-level structure, as discussed in Section 1.2, and the controlled-phase gate can be realized when the reflected photon re-scatters with the emitter. Besides, the re-scattering mechanism should be able to switched off, when that is not needed;
- In [2], single-qubit operations on photons are not required, however in our protocol to generate tree states, we need to apply Hadamard gates on photons, and in our protocol to generate RGSs, Hadamard gates and other single-qubit gates are applied on photons, so these are required to be easy to achieve;
- With the increase of the size of tree states (and RGSs), a long time-delayed feedback line is required, and this may be furthered limited by the pumping rate of the emitter. The longer the feedback line is, the more vulnerable the system is to photon loss, attenuation, and other factors.

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